Kuwait University, Department of Mathematics and Computer Science

Calculus I - MidTerm I- April 8, 2010- 90 min

Calculators and Phones are not allowed.

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1. (4pts) Evaluate the following limits, if they exist.

(i)

$$\lim_{x \to -\infty} x + \sqrt{x^2 + 3x + 1}.$$

(ii)

$$\lim_{x \to \pi} (x - \pi)^2 \cos\left(\frac{1}{\pi - x}\right).$$

2. (6pts) Let f be a function given by

$$f(x) = \begin{cases} \dfrac{x-5}{x^2 - 6x + 5} & \text{if } \ x < 0, \\[2ex] \dfrac{x^2 + x}{x^3 - x} & \text{if } \ 0 < x < 1, \\[2ex] \dfrac{-2|x-2|}{x^2 - 3x + 2} & \text{if } \ x > 1. \end{cases}$$

Find and classify the discontinuities of f.

3. (3pts) Determine $f'(\pi)$ for

$$f(x) = \sin(x)\tan(x) + \frac{3\cos(x)}{1 + \sin(x)}.$$

4. (3pts) At what point(s) on the curve $y = x\sqrt{x}$ is the tangent line parallel to the line $6x - 2y + 7 = 0$?

5. (3pts) Use the definition of the derivative to evaluate $f'(-1)$ for $f(x) = \sqrt{1 - x}$.

6. (3pts) Let $f(x) = x^2 - 2x$ and $g(x) = 1 - x^9$. Show that the graphs of f and g intersect.

7. (3pts) Find the vertical and horizontal asymptotes, if any, for

$$y = \frac{|x|(2x^2 + 3)}{x^3 + 8}.$$

1. (i) $\lim_{x\to-\infty} x+\sqrt{x^2+3x+1} = \lim_{x\to-\infty} \dfrac{(x+\sqrt{x^2+3x+1})(x-\sqrt{x^2+3x+1})}{(x-\sqrt{x^2+3x+1})}$

$\lim_{x\to-\infty} \dfrac{(-3x-1)}{(x-|x|\sqrt{1+\frac{3}{x}+\frac{1}{x^2}})} = \lim_{x\to-\infty} \dfrac{x(-3-\frac{1}{x})}{x(1+\sqrt{1+\frac{3}{x}+\frac{1}{x^2}})} = -\dfrac{3}{2}.$

(ii) $\lim_{x\to\pi} (x-\pi)^2 \cos\left(\dfrac{1}{\pi-x}\right) = 0.$ (Sandwich Theorem)

2. f is discontinuous at $x = 0, 1, 2$.

a. At $x = 0$: f has removable discontinuity.

$\lim_{x\to0^-} \dfrac{x-5}{(x-5)(x-1)} = -1.$

$\lim_{x\to0^+} \dfrac{x^2+x}{x^3-x} = \lim_{x\to0^+} \dfrac{x(x+1)}{x(x-1)(x+1)} = -1.$

b. At $x = 1$: f has infinite discontinuity.

$\lim_{x\to1^-} \dfrac{x(x+1)}{x(x-1)(x+1)} = -\infty.$

$\lim_{x\to1^+} \dfrac{-2|x-2|}{(x-2)(x-1)} = +\infty.$

c. At $x = 2$: f has jump discontinuity.

$\lim_{x\to2^-} \dfrac{2(x-2)}{(x-2)(x-1)} = 2.$

$\lim_{x\to2^+} \dfrac{-2(x-2)}{(x-2)(x-1)} = -2.$

3. $f'(x) = \cos(x)\tan(x) + \sec^2(x)\sin(x) + 3\left(\dfrac{-\sin(x)(1+\sin(x)) - \cos(x)\cos(x)}{(1+\sin(x))^2}\right).$

$f'(\pi) = -3$

4. $y = x\sqrt{x}$. Thus, $y' = \frac{3}{2}\sqrt{x}$. Setting $\frac{3}{2}\sqrt{x} = 3$, we get $x = 4, y = 8$.

5. $f'(-1) = \lim_{h\to0} \dfrac{\sqrt{2-h}-\sqrt{2}}{h} = \lim_{h\to0} \left(\dfrac{\sqrt{2-h}-\sqrt{2}}{h}\right)\left(\dfrac{\sqrt{2-h}+\sqrt{2}}{\sqrt{2-h}+\sqrt{2}}\right) =$
$-\dfrac{1}{2\sqrt{2}}.$

6. Let $h(x) = f(x) - g(x) = x^2 - 2x - 1 + x^9$. Since h is continuous everywhere, and $h(-1) = 1 > 0$ & $h(0) = -1 < 0$, then by IVT, $\exists\, c \in (-1, 0)$ such that $h(c) = 0$. Thus, the graphs of f and g intersect.

7. $\lim_{x\to\pm\infty} \dfrac{|x|(2x^2+3)}{x^3+8} = \pm2$. Thus, $y = \pm2$ (H.A).

$\lim_{x\to-2^\pm} \dfrac{|x|(2x^2+3)}{(x+2)(x^2-2x+4)} = \pm\infty$. Thus, $x = -2$ (V.A).